



Floquet Mode Circulation using a Coarsely Discretized Dielectric Huygens' Metasurface

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Abstract

This paper reports a *ternary dielectric Huygens' metasurface* – a metasurface comprising three elements per period, that performs *Floquet* mode circulation in k -space. We modeled the metasurface as a three-port network, and through full-wave simulation at 28 GHz, we show that the proposed metasurface performs circulation by transforming the FB modes from mode number $(+1, 0, -1)$ to $(0, -1, +1)$, respectively. The corresponding input and output angles are $(+33.8^\circ, 0^\circ, -33.8^\circ)$ and $(0^\circ, -33.8^\circ, +33.8^\circ)$, respectively.

1 Introduction

Electromagnetic metasurfaces are thin two-dimensional artificial structures that have sparked considerable interest as a new paradigm for manipulating electromagnetic (EM) waves in a nearly arbitrary manner [1]. Over the past few years, metasurfaces have been designed for a plethora of applications, including anomalous wave refraction and reflection [2–5], solving mathematical equations [6], among many others. Following the quest for complete wavefront control, *Huygens' metasurfaces* – performing wavefront transformation through the utilization of *Schelkunoff's equivalence principle*, have been proposed [7, 8]. Huygens' metasurface (HMS) elements can either be implemented using metallic scatterers [9] or dielectric resonators [10]. At high frequencies, such as millimeter-wave, terahertz, and beyond, the implementation of the former is challenging due to the requirement of vias or several metallic layers. Furthermore, the inherent joule losses reduce efficiency, preventing them from being used for high frequency applications.

Dielectric Huygens' metasurface (DHMS), on the other hand, has been suggested as a low-loss alternative to their metallic counterparts [10]. These artificial surfaces are composed of low-loss and high-index dielectric resonators, in which the interference between the electric and magnetic dipolar modes having comparable strength, enhances forward scattering and suppresses backward scattering, similar to the *Kerker's first condition*. By carefully tuning the

resonator geometry, the spectral overlapping between the eigenmodes can be realized in a single dielectric block, resulting in high transmission and full 2π phase coverage.

One of the most fundamental functions of the metasurface is to perform wave deflection through a phase-gradient approach based on generalized Snell's law [2]. Such metasurfaces are made up of densely packed subwavelength sized meta-atoms with spatially varying properties that redirect the impinging wave locally. However, these designs are often inefficient, particularly for large steering angles [3]. Later, this issue has been addressed by enforcing the exact boundary condition on the metasurface to implement high efficiency meta-devices [4]. Nevertheless, in order to implement rapidly varying surface impedance, such designs necessitate high resolution discretization.

To address the inadequacies of the design methodologies outlined above, the notion of *coarse discretization* has recently been proposed. A coarsely discretized metasurface consists of a single or a few polarizable particles per period and is capable of performing extreme wave manipulation with high efficiency and simplicity [11–13]. Such metasurfaces have a relatively larger unit-cell size, which reduces mutual interaction between the elements and simplifies fabrication. The concept of coarse discretization is based on long-established grating physics, in which the local period selects a discrete set of diffraction channels known as *Floquet-Bloch* (FB) modes – a series of propagating and evanescent plane waves excited by periodic structure. The meta-atoms in this scenario are engineered to steer the incident wave towards the intended Floquet channel direction while suppressing the spurious FB modes. When such metasurfaces are carefully designed with an appropriate discretization level, an FB mode circulation effect can be observed in the spectral or k -space domain.

In this paper, we present a *ternary dielectric Huygens' metasurface* – a metasurface composed of three elements per period, that performs anomalous refraction and realizes Floquet mode circulation in the spectral domain from mode index $m = +1, 0, -1$ to $m = 0, -1, +1$, respectively. In addition, we characterized the proposed metasurface as a

three-port system, with each port representing different FB mode selected by the metasurface.

2 Metasurface Discretization and k -space Circulation

Consider a periodic metasurface in free-space with period Λ_g and spatial frequency $k_g = \frac{2\pi}{\Lambda_g}$. Upon illumination by a plane wave, the metasurface scatters a discrete set of propagating and evanescent plane waves, generally known as *Floquet-Bloch* (FB) modes. The transverse spatial frequency of these FB modes is expressed as

$$k_{y(m)} = k_{yi} + mk_g = k_{yi} + mk_g, \quad (1)$$

where k_{yi} is the y -directed wave number of the incident wave and m ($= 0, \pm 1, \pm 2, \dots$) represents the FB mode number. Fig. 1 illustrates the spectral or k -space operation of the metasurface, which can be expressed mathematically as

$$\Omega_o(k_y) = \sum_m A_m \delta(k_y - k_{y(m)}) \quad (2)$$

where $\Omega_o(k_y)$ is the output k -space spectrum, A_m represents the amplitude, and $k_{y(m)}$ is given by (1). The output spectrum consists of an infinite number of FB modes, as shown by the purple arrows in Fig. 1. However, only a finite number of these modes which falls inside the propagation region with $k_y \in [-k_0, k_0]$ (represented by a purple box in Fig. 1) can diffract into the far-field, whereas the modes outside the aforementioned region are evanescent in nature and remain in the near field of the metasurface. It has been demonstrated in [12] that aggressively discretizing the metasurface with M -elements per period is sufficient to modulate the power in the M -FB modes that propagate into the far-field. When appropriately designed, such metasurfaces can exhibit the circulation property in the spectral domain, as will be discussed further below.

Consider a coarsely discretized periodic metasurface, which upon illumination by a plane wave generates $2P + 1$ -FB modes in the output spectrum, separated by $k_g = \frac{2\pi}{\Lambda_g}$. As a result of coarse discretization in the spatial domain, the metasurface shows the periodic character in the spectral domain. The whole k -space spectrum of the metasurface is periodic of these $2P + 1$ modes. Due to the periodic nature of the k -space spectrum, a coarsely discretized metasurface

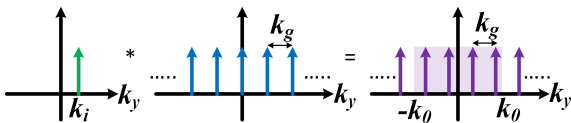


Figure 1. k -space operation of a periodic metasurface. Arrows denote possible spectral components' spatial frequencies but do not provide information about the amplitude and phase. The purple box indicates the propagation regime.

exhibits a mode circulation property [14]. For simplicity, we here consider the case of three propagating FB modes, *i.e.*, $P = 1$, where the metasurface refracts a normally incident wave towards an anomalous direction, and only the -1^{st} mode placed at $-k_g$ has non-zero amplitude. These three FB modes can be controlled by coarsely discretizing the metasurface with three elements per period [12–14]. The aforementioned discretization results in the periodicity of the k -space spectrum of the metasurface with a period of $3k_g$. Fig. 2(a) shows the k -space operation of the metasurface with spatial frequency $k_g \in (k_0/2, k_0)$. For this metasurface, an input wave with tangential wave number $k_{yi} = +k_g, 0, -k_g$ produces non-zero mode with tangential wave number $k_{yt} = 0, -k_g, +k_g$ within the propagation regime at the output spectrum (transmission spectrum), as depicted schematically in Fig. 2(b). As a proof of concept, we design a three-element transmissive DHMS to illustrate the mode circulation property.

3 Ternary Dielectric Huygens' Metasurface

We begin by examining the modes of a dielectric meta-atom¹ (refer to Fig. 3(a)) placed in an infinite array along the x - and y -directions. The simulation is carried out in Ansys HFSS with periodic boundary conditions and Floquet ports. The structure is illuminated by an x -polarized plane wave propagating along the $-z$ -axis. Fig. 3(b) shows the transmission spectra of an example meta-atom. The electric and magnetic dipolar modes are represented by the transmission magnitude minimas at 27.3 GHz and 29.9 GHz, respectively. This is further confirmed by the vector electric field distribution displayed in Fig. 3(c). Examining the electric field distribution at 27.3 GHz reveals the excitation of an electric dipole mode oriented along the x -direction (refer to the upper panel of Fig. 3(c)), whereas the anti-parallel electric field lines (refer to the lower panel of Fig. 3(c)) at 29.9 GHz reveals the excitation of a magnetic dipole along the y -direction. The aforementioned modes can then be brought into spectral overlap by appropriately adjusting the dimensions of the rectangular DR, resulting in high transmission and full 360° phase coverage, as shown in Fig. 3(d).

Next, we seek to design a transmissive DHMS that redirects a normal incident wave in an anomalous direction. Here, we consider the manipulation of the fundamental ($m = 0$) and the first-order ($m = \pm 1$) FB modes. The metasurface period should be set to $\lambda_0 < \Lambda_g < 2\lambda_0$ to ensure that the higher-order FB modes ($|m| \geq 2$) remain evanescent [12]. The transverse wavenumber of the first order Floquet modes ($m = \pm 1$) for the normal incidence can be expressed as $k_{y(\pm 1)} = \pm k_0 \sin \theta_{t(\pm)}$, where eq. (1) yields the transmitted angle, $\theta_{t(\pm 1)}$ as:

$$\theta_{t(\pm 1)} = \pm \sin^{-1} \left(\frac{\lambda_0}{\Lambda_g} \right) \quad (3)$$

¹The relative permittivity of the dielectric resonator is 12 and dielectric loss is not taken into account.

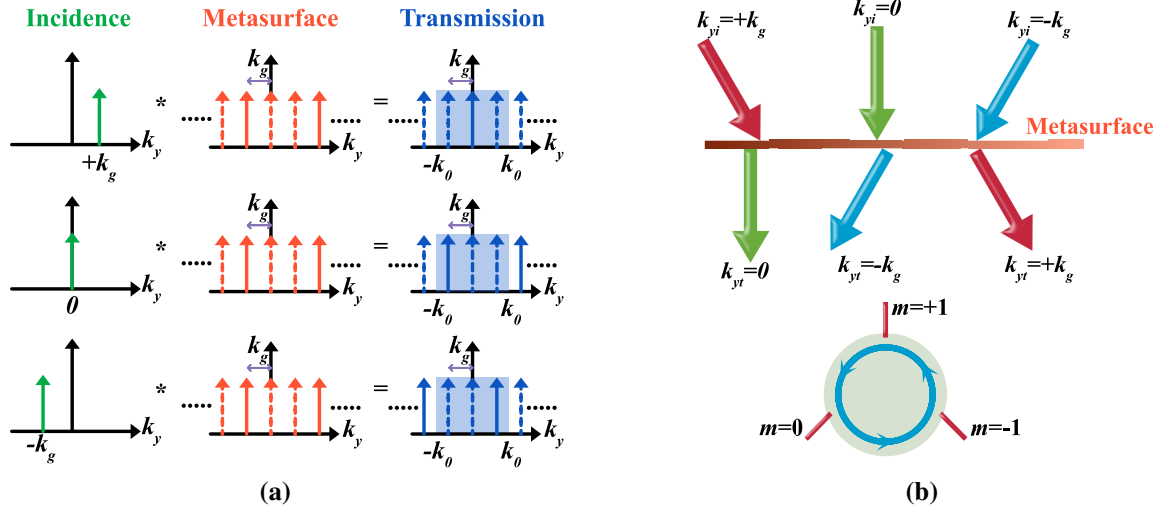


Figure 2. (a) The k -space operation of the periodic metasurface showing the circulation of three FB modes. Solid arrows represent FB modes with non-zero magnitude, while dashed arrows represent FB modes with zero magnitude. (b) A schematic of a transmissive metasurface that realizes the FB mode circulation with input mode number $m = +1, 0, -1$ to output mode number $m = 0, -1, +1$.

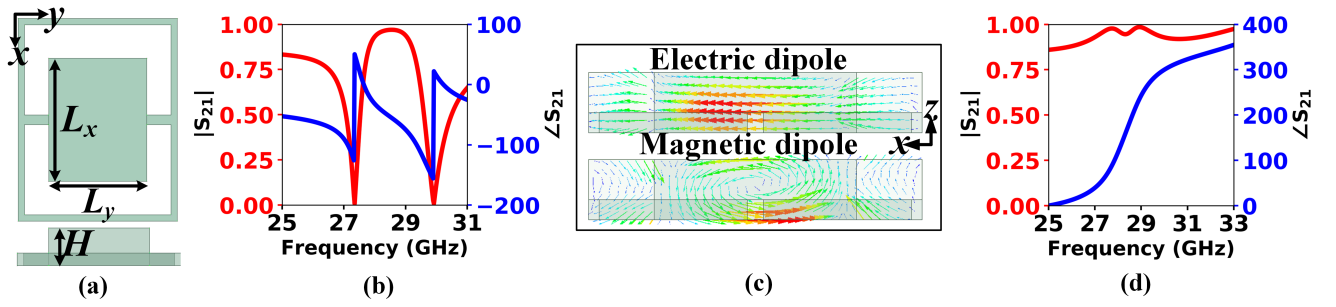


Figure 3. (a) Schematic of the dielectric meta-atom. (b) Transmission spectrum of an example dielectric meta-atom. (c) Electric field distribution at 27.3 GHz (upper panel) and 29.9 GHz (lower panel). (d) Transmission spectrum of the dielectric Huygens' metasurface, showing $|S_{21}| > 0.86$ and full 360° phase coverage across 25-33 GHz. The final optimized dimensions (in mm) are: $L_x = 5.5$, $L_y = 3.4$, $H = 1.7$.

For the transmissive metasurface, the above-mentioned modes are present in both the transmission and reflection regimes, adding up to a total of six FB modes. Fortunately, the metasurface based on Huygens' principle produces very weak reflections and can be considered as a near-reflectionless artificial surface [9]. Therefore, we only consider the three transmission modes, and a ternary discretization will suffice to alter the power carried by these transmission modes. Accordingly, we design the metasurface with three spatially varying dielectric meta-atoms per period, which we refer to as *ternary dielectric Huygens' metasurface* (T-DHMS). The periodicity of the metasurface is chosen as $\Lambda_g = 1.8\lambda_0$, where λ_0 is the free space wavelength determined at 28 GHz. The proposed metasurface redirects a normal incident wave towards the Floquet channel $m = -1$ that corresponds to $\theta_{(-1)} = -33.8^\circ$ according to (3).

To illustrate the mode circulation effect, we modeled the proposed metasurface as a three-port network, as depicted

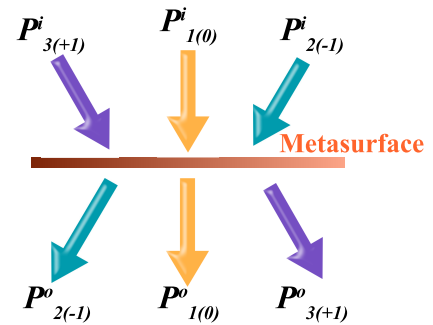


Figure 4. Schematic showing three-port model of the proposed metasurface.

in Fig. 4. Each input (incident) and output (transmission) port represents a different FB mode. The input ports are denoted as $P^i_{n(m)}$, while the output ports as $P^o_{n(m)}$, where $n = 1, 2, 3$ is the port number and $m = 0, -1, +1$ is the FB mode index. The related transmission coefficient (mag-

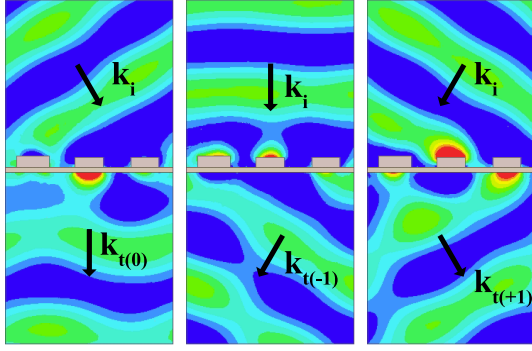


Figure 5. Electric field distribution in yz -plane at 28 GHz. Right panel shows the mode transformation from $m = +1$ to $m = 0$, middle panel shows the mode transformation from $m = 0$ to $m = -1$, and left panel depicts the mode transformation from $m = -1$ to $m = +1$.

nitude) matrix can be represented as t_{n_o, n_i} , where the wave is transmitted to n_o^{th} port when the metasurface is illuminated from n_i^{th} port. The transmission coefficient (magnitude) matrix of the proposed metasurface at 28 GHz is given by (4).

$$t_{n_o, n_i} = \begin{bmatrix} 0.27 & 0.30 & \mathbf{0.88} \\ \mathbf{0.9} & 0.25 & 0.33 \\ 0.27 & \mathbf{0.89} & 0.26 \end{bmatrix} \quad (4)$$

According to the transmission magnitude values (t_{13} , t_{21} , t_{32}), it is clear that the proposed metasurface realizes the mode circulation, transforming the input wave with mode numbers (+1, 0, -1) to the transmitted wave with mode numbers (0, -1, +1). The electric field distribution shown in Fig. 5 clearly displays the mode circulation property.

4 Conclusion

In summary, we have presented a theory and simple demonstration of Floquet modes circulation utilizing a simple, passive, and reciprocal dielectric Huygens' metasurface. The proposed coarsely discretized metasurface with a discretization level of three allows the FB mode to circulate from mode number (+1, 0, -1) to (0, -1, +1), respectively. Appropriately designing and discretizing the metasurface opens up new possibilities for designing circulators in both the transmission and reflection regimes.

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