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Modulation instabilities and group velocity dispersion in partially stripped magnetoplasma channels

B S Sharma[1], **D N Gupta**[2], **A Sharma**[3] **and A Jain**[1]

[1] Department of Physics, Govt. College Kota, Kota-324005, India
[2] Department of Physics and Astrophysics, University of Delhi, New Delhi-110 007, India
[3] Department of Electronics and Communication Engineering, Ambedkar Institute of Advanced Communication Technologies and Research, New Delhi-110031, India

E-mail: bssharma@physics.org

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Abstract

The nonlinear theory of propagation of an intense laser beam in a partially stripped magnetized plasma channel is investigated. An external magnetic field is considered, along the direction of propagation of the laser pulse. A three-dimensional envelope equation for the evolution of the laser field is derived. Using the source dependent expansion technique, an expression for the laser spot evolution is derived. The effect of the external magnetic field on the transverse oscillation of the laser spot is analyzed analytically and numerically. A formulation for the amplitude modulation instability is derived and the results are compared against the fully stripped plasma case. The excitation condition and the growth rate of the modulation instability are discussed. It is observed that increased positive chirp in the laser frequency enhances the modulation instability significantly. The group velocity dispersion is also calculated and the results are compared with those for the case of a fully stripped magnetized plasma channel. The presence of an external magnetic field and that of bound electrons have a significant effect on the dephasing length, as compared to that for a fully stripped plasma. The separability of the plasma frequency and laser frequency ($\omega_{\rm p}$ and ω_0 respectively) is considered, such that it satisfies the criterion $\omega_{\rm p}/\omega_0 < 1$.

Keywords: laser plasma interaction, partially stripped plasma, magnetized plasma channel, modulation instabilities, group velocity dispersion

(Some figures may appear in colour only in the online journal)

1. Introduction

The propagation of high intensity laser pulses in partially stripped plasma (PSP) and plasma channels is relevant to a wide range of applications such as in optical harmonic generators, x-ray sources, laser driven accelerators, and laser fusion [1–9]. It becomes more important when we deal with the interaction of a short intense laser pulse with a high atomic number target, such as in indirect laser fusion and x-ray lasers, where the ions in a plasma are only stripped partially. For example, for aurum target plasma, the ionization degree is 51 and the number of electrons remaining in each atom is 28 [10]. The partially stripped atoms become polarized in the presence of a very strong electric field of the laser and induce a nonlinear polarization current. Therefore, many characteristics of a laser pulse propagating in PSP undergo obvious changes [11–15]. The bound electrons affect the propagation of the laser energy in the plasma and can result in an atomic filamentation instability (AFI), which can dominate over the conventional relativistic filamentation instability [16–19], the atomic modulation instability (AMI), and the parametric instabilities. The mechanisms of these instabilities in PSP are quite different to those in the fully stripped plasmas (FSPs) [20]. It has been examined analytically that the free electrons generate anomalous group velocity dispersion (GVD), where the bound electrons result in self-phase modulation. This results in dispersive temporal broadening, which finally affects the propagation and the stability of the intense laser pulse. The atomic polarizability also changes the electric field profile of the laser pulse and the plasma wave. Sharma and Jain [21] have examined the effects of the polarizability on the wakefield generation of an intense laser pulse propagating in a partially stripped magnetoplasma channel. Recently, Sharma *et al* [22] have examined the laser pulse propagation in a parabolic magnetoplasma channel and wakefield generation. The analytical results were confirmed by particle-in-cell (PIC) simulations. In this paper, we build on the previous work by studying the modulation instability and GVD of a laser pulse in a magnetoplasma channel. The effect of the laser pulse propagation on the growth rate of the amplitude modulation instability in a partially stripped magnetoplasma channel is examined and a comparison with the case for a FSP is also presented in this work. Gupta and Suk [23, 24] have examined the excitation of a large amplitude plasma wave in a narrow band semiconductor in the presence of a wiggler magnetic field for two co-propagating laser beams without taking into consideration the effects of the concentrations of the free and bound charge carriers. And the effect of a magnetic field on the laser frequency chirping for a focused laser beam in an underdense plasma is used to analyze the response of the chirping to the resonance condition.

When the wave intensity is high enough, the polarization of the PSP becomes a nonlinear function of the wave field strength, so the AMI, AFI, self-phase modulation instability, and parametric instabilities start affecting the free charged particles and the bound electrons. Parametric instabilities such as those in stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) play an important role in transporting laser beam energy to the target. The growth rates of these instabilities [25, 26] are affected by the GVD, leading to a significant change in the stable propagation of the laser beam.

The preformed plasma channels created by an intense pulse are used for stable guiding of a short intense relativistic pulse [27, 28], to focus over several Rayleigh lengths without any change in the shape of the pulse. In inertial confinement fusion (ICF) experiments, the laser–hohlraum coupling depends on the smooth propagation of the laser pulse through the large scale gas-filled targets. For relativistic propagation through air or gases, the leading edge of the pulse creates an ionized plasma channel. It has been observed that an intense short laser pulse in such a preformed ionized plasma channel propagates over a greatly extended distance without any distortion in the shape and intensity [29]. Several other features of the propagation of short intense pulses in ionized plasma channels have been examined experimentally and through simulations [21, 30].

Contributions to the refractive index of an intense laser pulse in a PSP come from the bound atomic electrons arising from the finite spot size of the laser pulse $-2c/(\omega_0 r_0)^2$, the linear contribution from the free electrons of the plasma $-\omega_p^2/2\omega_0^2$, the nonlinear contribution from the excited plasma waves $\Delta\eta(\delta n)/n_0)$, the relativistic contribution from the plasma electrons, and the nonlinear contribution from the bound atomic electrons, where ω_0 is the laser frequency, r_0 is the laser spot size, $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ is the plasma frequency, n_0 is the ambient plasma density, δn is the perturbed plasma density, $a_0 = eA_0/mc^2$ is the normalized peak amplitude of the laser vector potential, and $\Delta\eta$ denotes the change in the refractive index of the medium. The response of these factors is different in the PSP channels when the channel is subjected to an external magnetic field in the direction of the pulse propagation. The different behaviors of these factors alter the characteristics of the pulse propagation, particularly the self-focusing. In addition to this, the presence of the bound electrons effectively changes the nonlinear regime when the electron plasma density is pushed to several times the critical density by the ponderomotive force associated with the intense laser beam [8], and reduces the effect of the Raman scattering. This may also lead to changes in the mechanism of the GVD, relativistic focusing, amplitude modulation instability, and other undesired parametric instabilities.

In this paper, we address the effects of an external magnetic field and the bound electrons on the evolution of the short laser pulse and related phenomena such as modulation instabilities, GVD, and the dephasing length. A nonparaxial theory is used to analyze the pulse evolution in the channel in the presence of an external magnetic field. The analysis is based on the assumption that no further ionization take place during the propagation of the pulse through the plasma channel. The response of the circularly polarized electromagnetic wave to the partially stripped magnetoplasma channel is different as compared to that for the FSP channels [21, 31]. A comparison of the two cases has been discussed in the results part. The effects of a constant external magnetic field on the self-focusing, GVD, and atomic modulation instability are underlined for a short intense circularly polarized Gaussian laser pulse propagating through a partially stripped magnetoplasma channel.

The present theoretical model may be useful for advanced laser fusion schemes [8], when the laser target source is situated at a large distance and the pulse needs to propagate through a preformed plasma channel. A nonlocal theory is used for obtaining the evolution of the laser pulse using the source dependent expansion (SDE) method. The effect of the external magnetic field on the spatial and temporal evolution of the pulse, GVD, and AMI are examined numerically.

This paper is organized as follows. In section 2, a three-dimensional wave equation for the evolution of the intense circular polarized Gaussian laser pulse in a partially stripped magnetoplasma channel is formulated. Solutions of the wave equation are deduced, along with the analytical description of the spatial evolution and the temporal evolution of the pulse. A relation for the growth rate of the modulation instability is derived and examined numerically in section 3. A theory for the GVD, GVD length, and dephasing length is formulated in section 4. The results are compared with the FSP case. A conclusion is given in section 5.

2. Laser envelope evolution

We consider the propagation of a circularly polarized laser pulse in a partially stripped parabolic plasma channel in the direction of the applied magnetic field B_{0z}. The amplitude $\boldsymbol{A}$ of the high frequency electromagnetic field can be expressed in terms of the vector potential $A(r, z, t)$ as

$$\boldsymbol{A}(r,z,t) = \frac{1}{\sqrt{2}}(\hat{e_x} + \mathrm{i}\hat{e_y})A(r,z)\mathrm{e}^{-r^2/r_{\mathrm{ch}}^2}\mathrm{e}^{-\left(\frac{t-z/v_g}{\tau_L}\right)^2} \mathrm{e}^{\mathrm{i}\left(\omega_0 t - \int_0^z k_0(z)\,\mathrm{d}z\right)} \tag{1}$$

where $\hat{e_x}$ and $\hat{e_y}$ are the unit vectors perpendicular to the guide magnetic field, $\nabla^2 = \nabla_\perp^2 + \partial^2/\partial z^2$, r is the effective transverse coordinate, z is the propagation direction, and $A(r, z)$ is assumed to be the axisymmetric amplitude of the high frequency field. In terms of the normalized vector potential $\boldsymbol{a}(= e\boldsymbol{A}/mc^2)$, the field equation is $\boldsymbol{a}(\boldsymbol{r}, t) = a(r, z)\mathrm{e}^{\mathrm{i}(\omega_0 t - \int_0^z k_0(z)\,\mathrm{d}z)}$ where $a(r, z) = a_{0,0}\mathrm{e}^{(-r/r_{\mathrm{ch}}(z))^2}\mathrm{e}^{-(t-z/v_g)^2/\tau_L^2}$, $a_{0,0}$ is the peak value of the normalized vector potential at the entrance of the channel ($z = 0, t = 0$), r_{ch} is the channel radius, and v_g is the group velocity of the laser pulse. For axis symmetric interaction, we have $\boldsymbol{\nabla}_\perp^2 \to r^{-1}\partial/\partial r + \partial^2/\partial r^2$.

For the study, the numerical parameters used are as follows: laser pulse duration $\tau_L \sim 2$ fs, intensity $I \leqslant 10^{15}$ W cm^{-2}, laser pulse wavelength $\lambda_0 = 0.5\,\mu$m, peak value of the normalized amplitude of the field $a_{0,0} \leqslant 0.05$, plasma density $n_0 \geqslant 10^{18}$ cm^{-3}, laser beam spot size at the entrance of the channel 6.4 μm; also, $\omega_{\mathrm{p}} = (4\pi n_0 e^2/m)^{\frac{1}{2}}$ is the on-axis plasma electron frequency at $r = 0$, where m and e are the mass and charge of the electron, and $k_{\mathrm{p}} = \omega_{\mathrm{p}}/c$. The plasma channel radius is chosen to vary in this way so that, in the absence of instability, the pulse and channel are matched. The plasma channel is characterized by a spatially varying density given as

$$n(r,z) = n(z)\left(1 + \frac{\Delta n}{n_0}\frac{r^2}{r_{\mathrm{ch}}^2(z)}\right), \tag{2}$$

where $r_{\mathrm{ch}}(z)$ and $n(z)$ denote the longitudinally varying channel radius and on-axis plasma density, respectively, and the channel depth $\Delta n = 1 \times 10^{16}$ cm^{-3}.

The critical power for relativistic self-focusing in a plasma [34, 35] and nonlinear focusing in a gas [26, 33, 36–37] are given by, respectively,

$$P_{\mathrm{p}} = \frac{2m^2c^5\omega_0^2}{e^2\omega_{\mathrm{p}}^2(z')}, \tag{3a}$$

and

$$P_a = \frac{c^2}{2\omega_0^2\eta_0\eta_{nl}}. \tag{3b}$$

The ratio of the critical powers is

$$R = \frac{P_{\mathrm{p}}}{P_a} = \frac{8\pi\chi^{(3)}m^2c^2\omega_0^4\eta_0}{e^2\omega_{\mathrm{p}}^2(z')}, \tag{4}$$

where $\eta_0 \cong 1$ for the bound (atomic) electrons and $\chi^{(3)}$ is the third-order susceptibility associated with the bound electrons, and its magnitude in esu is about $10^{-38}n_a$ cm^3 [25], where n_a is the density of partially stripped atoms. We consider values of $\chi^{(3)}$ for partially stripped atoms of the same order of magnitude as those for the neutral atom. The nonlinear value of $\chi^{(3)}$ for an ionized atom can be estimated if the charge state is small compared to the atomic number. Our results are based on the effective value of R, which is proportional to $\chi^{(3)}$ for partially stripped plasma. In the PSP, if $R \gg 1$, the bound electrons have a much greater influence on the focusing of the laser pulse than the free electrons. In our present example, we find that the ratio R is $\approx$700. It is of interest to note that the optimum power level required for initiating the laser fusion process can be maintained more efficiently in magnetized PSPs.

To study the dynamics of the nonparaxial propagation of the electromagnetic wave in a partially stripped magnetoplasma channel, we follow the approach used by Sharma and Jain [21]. The nonlinear dispersion relation, in terms of the ratio of the critical powers (R), for the laser pulse in a partially stripped magnetoplasma channel is given as

$$k_0(z') \approx \left(\frac{\omega_0^2}{c^2} - \frac{2}{r_0^2} - \frac{\omega_{\mathrm{p}}^2(z')\omega_0}{c^2(\omega_0 - \omega_{\mathrm{c}})} + \frac{8R\omega_0(\omega_0 - \omega_{\mathrm{c}})}{c^2} \mid a \mid^2\right)^{1/2}. \tag{5}$$

Following Sprangle *et al* [26], we have

$$\left[\nabla_\perp^2 + \frac{2\mathrm{i}v_g(z')\omega_0}{c^2}\frac{\partial}{\partial z'}\right]a(r,,\xi,z') = \frac{\omega_{\mathrm{p}}^2(z)\omega_0}{c^2(\omega_0-\omega_{\mathrm{c}})} \times\left(\frac{r^2}{r_{ch}^2(z')} - \frac{\omega_0}{\omega_{\mathrm{c}}} + \frac{\Delta n(z')}{n_0} - \frac{\mid a \mid^2}{2}\right) - \left[\frac{2}{r_0^2} + (1-\beta_g^2)\frac{\partial^2}{\partial z'^2}\right]a(r,\xi,z'). \tag{6}$$

Equation (6) implies that the atomic polarizability participates in the longitudinal self-modulation and the stationary self-focusing. The reason is that the longitudinal electron density perturbation contributes significantly for the group velocity $v_g(z')$.

We use the source dependent expansion (SDE) technique [38] to solve equation (6). The normalized amplitude of the pulse is given as

$$a(r,\xi,z') = a_{0,0}(z')\mathrm{e}^{-(r^2/r'^2(z'))} \times\mathrm{e}^{-(t-z/v_g)^2/\tau^2(z')} \times \psi(r,z'), \tag{7}$$

where $\psi(r, z') = \phi(z') \times \mathrm{e}^{-\mathrm{i}(\mu(z')r^2/r'^2)}$ is the overall phase factor which can be considered as a product of the laser pulse phase ($\phi(z')$) and a phase factor that arises due to mismatch of the tapered channel radius and the spot size $r'(z')$, $\tau(z')$ is the pulse duration at spatial coordinate z', and $\mu(z')$ is a dimensionless parameter that defines the inverse of the radius of curvature of the wavefront of the pulse.

In the SDE technique, the wave equation can be written as

$$\left[\nabla_\perp^2 + \frac{2\mathrm{i}v_g(z')\omega_0}{c^2}\frac{\partial}{\partial z'}\right]\boldsymbol{a}(r,\xi,z') = \boldsymbol{S}(r,\xi,z'), \tag{8}$$

where the source term $S(r, \xi, z')$ is given as

$$\boldsymbol{S}(r,\xi,z') = \frac{\omega_{\mathrm{p}}^2(z')\omega_0}{c^2(\omega_0-\omega_{\mathrm{c}})}\left[\left(\frac{r^2}{r_{\mathrm{ch}}^2(z')} - \frac{\omega_0}{\omega_{\mathrm{c}}} + \frac{\Delta n}{n_0} - \frac{\mid a \mid^2}{2}\right) - \left(\frac{2}{r_0^2} + (1-\beta_{\mathrm{g}}^2)\left(\frac{-2}{\tau(z')} + \frac{4\xi^2}{v_{\mathrm{g}}^2(z')\tau^2(z')}\right)\right)\right]\boldsymbol{a}(r,z',\xi). \tag{9}$$

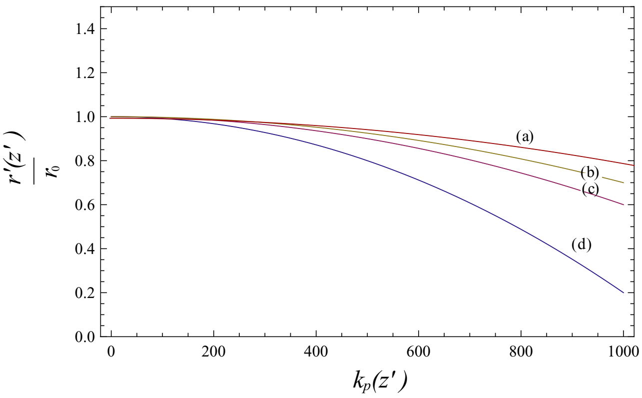

Figure 1. Variation of the normalized spot radius $r'(z')/r_0$ as a function of $k_\mathrm{p}(z')$ for different magnetic field ratios $\omega_\mathrm{c}/\omega_\mathrm{p}$: (*a*) 0.0, (*b*) 0.5, (*c*) 1.0, (*d*) 1.5.

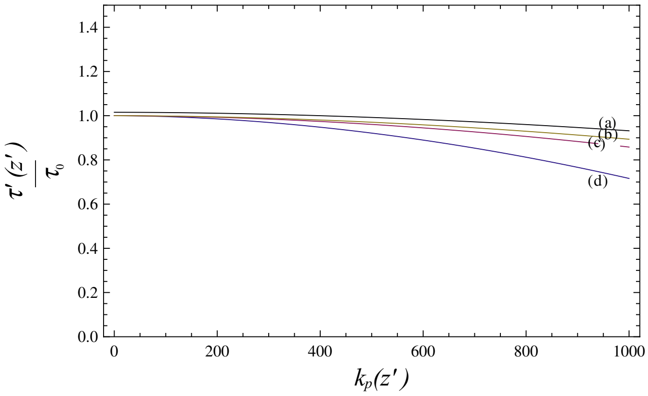

Figure 2. Variation of the normalized pulse duration $\tau'(z')/\tau_0$ as a function of $k_\mathrm{p}(z')$ for different magnetic field ratios $\omega_\mathrm{c}/\omega_\mathrm{p}$: (*a*) 0.0, (*b*) 0.5, (*c*) 1.0, (*d*) 1.5.

Using the standard SDE procedure, the equation governing the laser spot size $r'(z')$ and the pulse duration can be given by

$$\frac{\partial^2 r'(z')}{\partial z'^2} = \frac{4}{r'^3(z')k_0^2(z')} - \frac{\omega_\mathrm{p}^2(z)}{\omega_0(\omega_0-\omega_\mathrm{c})} \times \frac{\Delta n}{n_0}\left(1+\frac{\delta n}{n_0}\right)\frac{r'(z')}{r_{\mathrm{ch}}^{(}z')} - \frac{\omega_\mathrm{p}^2(z')}{\omega_0(\omega_0-\omega_\mathrm{c})}\frac{a^2}{2r'(z')}, \tag{10}$$

and

$$\frac{\partial^2 \tau(z')}{\partial z'^2} \backsimeq \frac{4}{\omega_0^2\tau^3(z')r_0^2} + \frac{k_0(z')\omega_\mathrm{p}^3}{c\omega_0^2(\omega_0-\omega_\mathrm{c})} \times \left(1+\frac{\Delta n}{n_0}\frac{r'^2(z')}{2r_{ch}(z')^2}\right)\frac{a^2(z')}{\tau(z')}. \tag{11}$$

The terms on the right-hand side of equation (10) include the effects of diffraction, plasma channel depth, external magnetic field, and plasma electron perturbation along the direction of pulse propagation, and the relativistic effect, respectively. The first and second terms on the right-hand side of equation (11) are responsible for the GVD effect, channel profile effect, and the nonlinear relativistic effect in the presence of the external magnetic field. The spatial evolution of the phase parameters and the other dimensionless parameter $\mu(z')$ are given by the following equations:

$$\frac{\partial a(z')}{\partial z'} = \frac{a(z')}{r'(z')}\frac{\partial r'(z')}{\partial z'} - \frac{a(z')}{2\tau(z')}\frac{\partial \tau(z')}{\partial z'}, \tag{12}$$

$$\frac{\partial \psi(z')}{\partial z'} = \frac{-2}{k_0(z')r'^2(z')} + \frac{2}{k_0(z')r_0^2} + \frac{1}{\tau^2(z')\omega_0(\omega_0-\omega_\mathrm{c})}\frac{-k_0(z')\omega_\mathrm{p}^2(z')}{\omega_0(\omega_0-\omega_\mathrm{c})} \times \left[-\frac{a^2}{8}\left(1+\frac{\Delta n(z')}{n_0}\frac{r'^2(z')}{2r_{\mathrm{ch}}^2(z')}\right)\right], \tag{13}$$

$$\mu(z') = \frac{-1}{2}k_0(z')r'(z')\frac{\partial \tau(z')}{\partial z'}. \tag{14}$$

Equations (9)–(14) represent the evolution of the various parameters of the laser pulse in a magnetoplasma channel under the condition that the ponderomotive force does not distort the plasma channel.

Figure 1 shows the variation of the normalized radius $(r'(z')/r_0)$ of the laser spot with $k_\mathrm{p}(z')$ for different values of $\omega_\mathrm{c}/\omega_\mathrm{p}$. The example is studied in the range of magnetic field from $B_0 = 10^3$ T ($\omega_\mathrm{c} = 17.8\times10^{13}$ rad s^{-1}) to $B_0 = 7\times10^3$ T ($\omega_\mathrm{c} = 12.5\times10^{14}$ rad s^{-1}). Although a magnetic field of this order cannot be generated in a laboratory, a quasistatic laser generated magnetic field of this magnitude or higher can in some cases be supported [39]. It is observed that the laser spot size decreases (pulse compression) with increasing $\omega_\mathrm{c}/\omega_\mathrm{p}$. The presence of the polarizability term adds a significant contribution to the pulse compression, and hence enhances the laser power to the significant level required to initiate the laser fusion process. Figure 2 shows the variation of the normalized time period $(\tau'(z')/\tau_0)$ of the pulse as a function of $k_\mathrm{p}(z')$ for different magnetic field ratios $\omega_\mathrm{c}/\omega_0$. It is observed that the pulse duration decreases upon increasing the external magnetic field and the laser pulse becomes more positively chirped.

3. Modulation instability

When an intense laser pulse propagates in a partially stripped magnetoplasma channel, the local phase velocity v_p and the group velocity v_g change, which can induce many nonlinear phenomena such as longitudinal bunching, transverse focusing, and photon acceleration. In PSP channels, the phase and the group velocity are different to those of the FSP channel. The presence of an external magnetic field further alters the mechanism of nonlinear interaction of the pulse with the channel. This leads to significant changes in the growth of the modulation instability. The group and the phase velocity can be determined from the wave equations. Following Decker and Mori [40, 41], the group velocity of the laser pulse in the PSP channel is given by

$$v_\mathrm{g} = c\left[1 - \frac{c^2}{r_0^2\omega_0^2} - \frac{\omega_\mathrm{p}^2(z')}{\omega_0(\omega_0-\omega_\mathrm{c})}\left(1+\frac{\alpha\omega_0(\omega_0-\omega_\mathrm{c})}{\omega_\mathrm{p}^2(z')}\right)\right], \tag{15}$$

where $\alpha = 8R|a|^2$. We assume that the laser frequency ω_0 changes to $\omega + \partial\omega_0$ when the pulse centroid moves over one plasma wavelength. Expanding the group velocity and neglecting the product of the perturbed quantities, we get

$$v_g \approx c\Big[1 - \frac{c^2}{r_0^2\omega^2}\left(1 - \frac{2\partial\omega_0}{\omega}\right) - \frac{\omega_p^2}{\omega(\omega-\omega_c)}\left(1 + \frac{\delta n}{n_0} - \frac{\partial\omega_0}{\omega} - \frac{\langle a^2(z')\rangle}{2}\right) - \alpha\left(1 - \frac{\partial\langle a^2(z')\rangle}{\langle a^2(z')\rangle} - \frac{\partial\omega_0}{\omega}\right)\Big], \quad (16)$$

where we have considered a spatially nonlocalized perturbation in the field intensity, $\langle a^2\rangle + \partial\langle a^2\rangle$, where the symbol $\langle\ \rangle$ refers to the average value of a^2. Similarly, the phase velocity,

$$v_p = c\left(1 + \frac{c^2}{r_0^2\omega_0^2} + \frac{\omega_p^2(z')}{\omega_0(\omega_0-\omega_c)} - \alpha\frac{(\omega_0-\omega_c)}{\omega_0}\right) \quad (17)$$

turns into the following form:

$$v_p = c\left[1 + \frac{c^2}{\omega^2 r_0^2}\left(1 - 2\frac{\partial\omega_0}{\omega}\right) + \frac{\omega_p^2}{\omega(\omega-\omega_c)} \times\left(1 + \frac{\delta n}{n_0} - \frac{\partial\omega_0}{\omega} - \frac{\langle a^2\rangle}{2}\right)\frac{-\alpha(\omega-\omega_c)}{\omega_p}\left(1 - \frac{\partial\omega_0}{\omega}\right)\right]. \quad (18)$$

We assume that there is no further ionization and recombination during the pulse propagation in the channel. So, within a local volume, the total photon number i.e. the classical action, is conserved. Following Mori [42], the vector potential of the laser can be modulated in three ways: modulate the longitudinal extent then induce longitudinal bunching; modulate the spot size then induce transverse focusing; modulate the pulse frequency then induce photon acceleration. All these three parameters change during the pulse propagation in the channel. In the present discussion, our focus is on the longitudinal bunching and the photon acceleration, closely related to the modulation instability. The longitudinal bunching and the photon acceleration, in the speed of light frame variables $\xi = t - z/c$ and $\tau = t$, can be understood as [42]

$$\frac{1}{l}\frac{\partial l}{\partial\tau} = \frac{1}{c}\frac{\partial v_g}{\partial\xi} \quad (19a)$$

and

$$\frac{1}{\omega_0}\frac{\partial\omega_0}{\partial\tau} = -\frac{1}{c}\frac{\partial v_p}{\partial\xi}. \quad (19b)$$

Assuming that the change of the intensity is only caused by the change in the longitudinal extent due to different nonlocal group velocity, $\delta\langle a^2\rangle$ then evolves with τ as

$$\frac{\partial(\delta\langle a^2\rangle)}{\partial\tau} = \frac{1}{c}\frac{\partial v_g}{\partial\xi}\langle a^2\rangle. \quad (20)$$

The change in the group velocity is mainly due to the change in the pulse frequency ω_0, so substituting equation (16) into equation (20) results in

$$\frac{\partial(\delta\langle a^2\rangle)}{\partial\tau} = \left(\frac{2c^2}{r_0^2\omega_0^2} - \frac{\omega_p^2}{\omega(\omega-\omega_c)} - 4Ra_{0,0}^2\right)\frac{c}{\omega}\frac{\partial\omega_0}{\partial\xi}, \quad (21)$$

where $\langle a^2\rangle = a_{0,0}^2/2$. Equation (21) shows that the partially stripped atoms reduce the longitudinal bunching of the energy. The derivative of equation (21) with respect to τ is

$$\frac{\partial^2(\delta\langle a^2\rangle)}{\partial\tau^2} = \left(\frac{2c^2}{r_0^2\omega_0^2} - \frac{\omega_p^2}{\omega(\omega-\omega_c)} - 4Ra_{0,0}^2\right) \times\frac{c}{\omega}\frac{\partial}{\partial\xi}\frac{\partial\omega_0}{\partial\tau}. \quad (22)$$

From equation (17) and equation (19*b*), we get

$$\frac{1}{\omega}\frac{\partial(\delta\omega_0)}{\partial\tau} \approx -\left(\frac{\omega_p^2}{\omega(\omega-\omega_c)} + 8R\right)a_{0,0}^2 c\frac{\partial\langle a^2\rangle}{2\partial\xi}. \quad (23)$$

Equation (23) shows that in the front edge of the perturbation, $(1/\omega)(\partial\delta\omega_0/\partial\tau) < 0$, the frequency is decreasing, while in the back edge of the perturbation, $(1/\omega)(\partial(\delta\omega_0)/\partial\tau) > 0$, the frequency is increasing, i.e., the perturbation is positively chirped. Further, the terms R and ω_c in equation (23) make the left-hand side term $(1/\omega)(\partial(\delta\omega_0)/\partial\tau)$ more positive. Thus it is anticipated that the presence of bound electrons and that of an external magnetic field give rise to a significant enhancement of the positive chirp. If $\delta\langle a^2\rangle = \langle a^2\rangle_1 e^{i(\omega_1 t - \int_0^z k_1(z)\,dz)} \approx \langle a^2\rangle_1 e^{-ik_1(z)c\xi}$, then

$$\frac{\partial^2\delta\langle a^2\rangle_1}{\partial\tau^2} \approx \frac{\omega_p^2 c^4 a_{0,0}^2 k_1^2(z)}{\omega^3 r_0^2(\omega-\omega_c)}\left(1 - \frac{2Ra_{0,0}^2 r_0^2\omega^2}{c^2}\right) \times\left(1 + \frac{8R\omega^2}{\omega_p^2}\right)\langle a^2\rangle_1. \quad (24)$$

Following [42], we add $-(1/4)(k_1^4\omega_p^4 c^4/\omega^6)\langle a^2\rangle_1$ to the right-hand side of equation (24) and obtain the final equation for the phase modulation instability as

$$\frac{\partial^2\delta\langle a^2\rangle_1}{\partial\tau^2} \approx \frac{\bar{k}_1^2}{4\bar{\omega}_1^{\,2}}\left[\frac{\omega c^2 a_{0,0}^2}{r_0^2(\omega-\omega_c)}\left(1 - \frac{2Ra_{0,0}^2 r_0^2\omega^2}{c^2}\right) \times\left(1 + 8R\bar{\omega}_1^{\,2}\right) - \bar{k}_1^{\,2}\right]\langle a^2\rangle_1 \quad (25)$$

where $\bar{k}_1 = k_1/k$ and $\bar{\omega}_1 = \omega_1/\omega_p$, and $k = \omega/c$. The modulation instability is excited only if $\partial^2\langle a^2\rangle_1/\partial\tau^2 > 0$. From equation (25), we have the following excitation condition for the modulation instability:

$$\frac{\bar{k}_1^2}{4\bar{\omega}_1^{\,2}}\left[\frac{\omega a_{0,0}^2 c^2}{r_0^2(\omega-\omega_c)}\left(1 - \frac{2Ra_{0,0}^2 r_0^2\omega^2}{c^2}\right) \times\left(1 + 8R\bar{\omega}_1^{\,2}\right) - \bar{k}_1^{\,2}\right]\langle a^2\rangle_1 > 0. \quad (26)$$

If equation (25) is satisfied, the growth rate of the modulation instability can be given as

$$\Gamma_{AM} = \left[\frac{\bar{k}_1^{\,2}}{4\bar{\omega}_1^{\,2}}\left[\frac{\omega a_{0,0}^2 c^2}{r_0^2(\omega-\omega_c)}\left(1 - \frac{2Ra_{0,0}^2 r_0^2\omega^2}{c^2}\right) \times\left(1 + 8R\bar{\omega}_1^{\,2}\right) - \bar{k}_1^{\,2}\right]\right]^{1/2}. \quad (27)$$

The stability boundary condition (equation (25)) is plotted in figure 3 for $R = 700$ and various normalized amplitudes $a_{0,0}$: (*a*) 0.01, (*b*) 0.02, (*c*) 0.035, and (*d*) 0.05, for magnetic field ratios $\omega_c/\omega_p = 0.0$ and $\omega_c/\omega_p = 1.5$. The plots are drawn for ω_0/ω_p versus k_1/k. The instability excited region is shown

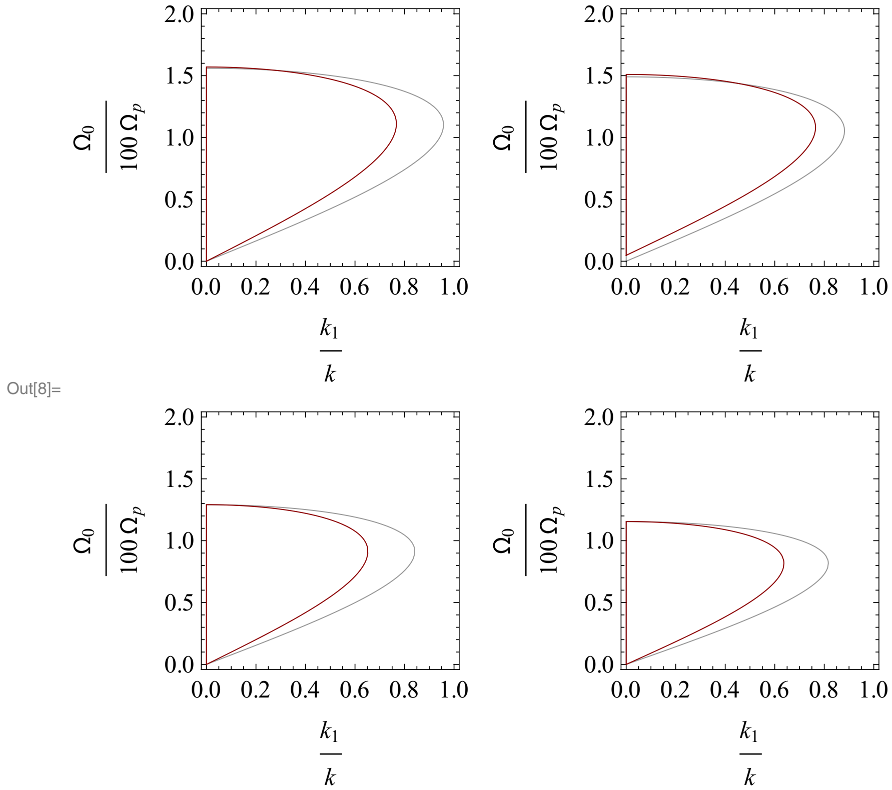

Figure 3. Stability boundary condition for the following values of $a_{0,0}$: (*a*) 0.01, (*b*) 0.02, (*c*) 0.035, (*d*) 0.05. The instability region is on the left side of the curve. Variations for $\omega_c = 0$ and $\omega_c/\omega_p = 1.5$ are shown by red and gray curves respectively.

on the left side of the curve. It is noted that the instability region decreases as the intensity of the radiation increases, and depends on the cyclotron frequency ω_c. It can be seen from equation (27) that on increasing the magnetic field ratio ω_c/ω_p from zero to 1.5, the instability excited region increases. This effect of the magnetic field is shown in figure 3. Here, the red curve is for $\omega_c = 0$ and the gray one for $\omega_c/\omega_p = 1.5$. The finite instability region may arise as a result of the finite-perturbation-length effect. Here, it is obvious that the finite region of instability is due to the competition between the normal and abnormal dispersions. The PSP acts as a normal dispersion medium, where the group velocity decreases with increase of the frequency. The FSP acts as an abnormal dispersion medium, where the group velocity increases with increase of the frequency. When the abnormal dispersion dominates over the normal dispersion, i.e., $8R+\Delta < 1$, where $\Delta = 4r_0^2(\omega - \omega_c)/\omega_0 a_{0,0}^2 c$, the positive chirp compresses the perturbation and the perturbation intensity increases; then the instability can be excited. When the normal dispersion dominates over the normal dispersion, i.e., $8R + \Delta > 1$, then the positive chirp disperses the perturbation and the perturbation decreases; then the instability cannot be excited. Similarly, when the normal dispersion is approximately equal to the abnormal dispersion, i.e., $8R + \Delta = 1$, the perturbation will be in its original state, and a soliton may be present.

The ratio of the growth rate of the modulation instability Γ_{AM} to the maximum growth rate of the forward Raman scattering (FRS) instability is $\Gamma_{FRS} = a_{0,0}\omega_p^2/2\sqrt{2}\omega$ [42], i.e.,

$$\frac{\Gamma_{AM}}{\Gamma_{FRS}} = \left[\frac{\bar{k_1}^2}{\sqrt{2}}\left[\frac{a_{0,0}c^2}{r_0^2(\omega-\omega_c)}\left(1-\frac{2Ra_{0,0}^2 r_0^2\omega^2}{c^2}\right)\right.\right.$$
$$\left.\left.\times\left(1+8R\bar{\omega}_1^{\,2}\right)-\bar{k_1}^2\right]\right]^{1/2}. \qquad (28)$$

In figure 4, we have plotted the instability growth rate ratio Γ_{AM}/Γ_{FRS} as a function of the wavevector ratio k_1/k for different magnetic field ratios ω_c/ω_p. It is observed that $\Gamma_{AM}/\Gamma_{FRS} > 1$ for a given ratio of the magnetic field. The instability ratio for the partially stripped magnetoplasma channel is greater as compared to that for the FSP channel. It is observed that the instability ratio is sensitive to the choice of magnetic field ratio and the polarizability of the bound electrons. In figure 4, we show the calculated modulation and Raman instability growth rates for $\omega_0/\omega_p = 5$, $\omega_c = 0.01\omega_0$, $R = 700$, and $a_{0,0} = 0.01$. The FRS instability growth rate Γ_{FRS} at $k_1/k = 6$ turns out to be about $7 \times 10^{10}\,\mathrm{s}^{-1}$. The growth of the amplitude modulation instability Γ_{AMI} for the same set of parameters turns out to be about $7.7 \times 10^{10}\,\mathrm{s}^{-1}$.

4. Group velocity dispersion

The GVD is responsible for the dispersive temporal broadening or compression of the ultrashort pulse. The perturbation in the pulse length leads to significant change in the process of self-phase modulation, relativistic effects, and GVD [43]. The pulse duration decreases with the magnitude of the GVD and the shortest pulses are generated at a point where the laser becomes unstable. For stable propagation over many Rayleigh lengths, the GVD must be positive. The presence of bound electrons and of the external magnetic field together intensify the positive chirp of the intensity perturbation significantly. This can be attributed to a significant change in the GVD and the relativistic effects. The combined effect of the GVD and relativistic nonlinear instabilities can modulate the propagation of an intense laser pulse over several Rayleigh lengths. In this section, a relation for the GVD in partially stripped magnetoplasma is derived.

Using equation (5), the GVD $(\partial^2 k_0(z)/\partial\omega_0{}^2)$ for a short laser pulse in a partially stripped magnetoplasma channel under matched conditions can be given as

$$\mathrm{GVD} = (\omega_0 - \omega_c)^{3/2}\left(\frac{4\omega_0^{1/2}}{c\omega_p^3} + \frac{16\alpha\omega_0^{5/2}}{\omega_p^3} + \frac{4}{\omega_p c(\omega_0-\omega_c)\omega_0^{1/2}} + \frac{8\alpha\omega_0^{3/2}c}{\omega_p(\omega_0-\omega_c)}\right). \tag{29}$$

It can be seen that the GVD is more positive in the PSP as compared to the FSP, due to the bound electrons. The presence of the external magnetic field also makes the GVD more positive and supports stable and extended propagation in plasma channels. This helps with attaining the desired power level for the laser–plasma fusion more efficiently than for the FSP.

For a stable propagation, the pulse dispersion length (Z_{GVD}) can be made more positive by modulating the pulse frequency, intensity of the laser radiation, channel radius, and plasma wavelength. The presence of bound electrons and of the external magnetic field make an effective change as compared to the FSP case.

The pulse dispersion length Z_{GVD} is given as

$$Z_{\mathrm{GVD}} = \frac{\omega_0^2 c^2}{\pi\omega_p^2}\mathrm{GVD} \approx \frac{\omega_0^2 c^2}{\pi\omega_p^2}(\omega_0-\omega_c)^{3/2} \times\left(\frac{4\omega_0^{1/2}}{c\omega_p^3} + \frac{16\alpha\omega_0^{5/2}}{\omega_p^3} + \frac{4}{\omega_p c(\omega_0-\omega_c)\omega_0^{1/2}} + \frac{8\alpha\omega_0^{3/2}c}{\omega_p(\omega_0-\omega_c)}\right). \tag{30}$$

The dephasing length can be given as

$$L_d = \frac{2\pi c^3}{\omega_p^2}\left(\frac{\omega_0^2}{c^2} - \frac{2}{r_0^2} + \frac{\omega_p^2\omega_0}{c^2(\omega_0-\omega_c)} + \frac{64\pi\omega_0^4 m^2\chi^{(3)}\mid a\mid^2}{e^2 c}\right). \tag{31}$$

Equation (31) implies that the presence of bound electrons in the PSP results in enhancement of the dephasing length as compared to the FSP case.

Figure 5 shows a comparative study of the variation of the dephasing length with the external magnetic field for the PSP and the FSP. Here, we numerically solve equation (31) with and without the last term in order to study the behavior of L_d with an external magnetic field for a PSP and a FSP,

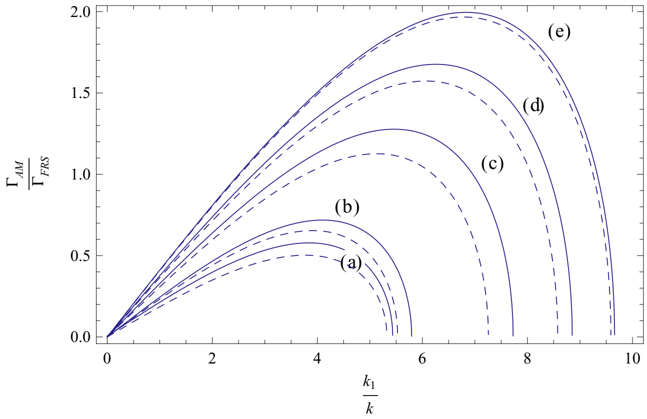

Figure 4. Variation of the instability growth rate ratio $\Gamma_{\mathrm{AM}}/\Gamma_{\mathrm{FRS}}$ with k_1/k for different magnetic field ratios ω_c/ω_p: (*a*) 0.0, (*b*) 0.5, (*c*) 1.0, (*d*) 1.5, (*e*) 2.0. The solid curves are for the PSP and the dashed curves for the FSP.

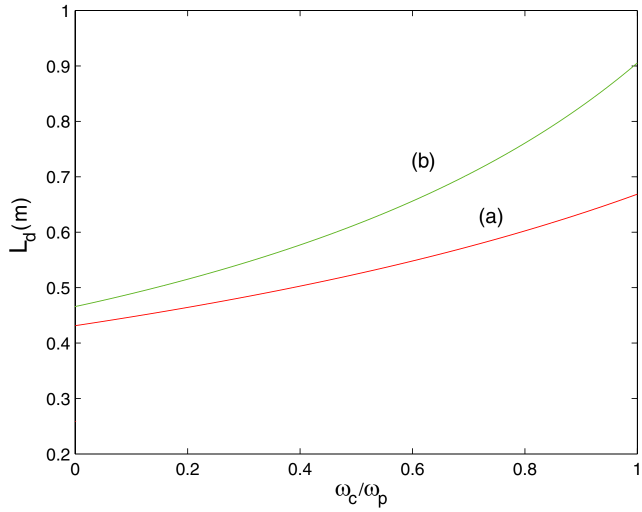

Figure 5. Variation of the dephasing length (L_d) with the magnetic field ratio ω_c/ω_p for $\omega_p = 2.8 \times 10^{13}$ rad s^{-1} for (*a*) FSP and (*b*) PSP. At $\omega_c = 0$, L_d for PSP is greater than that for FSP.

respectively. It is noted that the dephasing length L_d increases with increasing magnetic field ratio ω_c/ω_p more strongly for a PSP than for a FSP. The rate of increase of the dephasing length in the PSP is greater than that for the FSP for a given magnetic field. This suggests that the polarization current of the bound electrons plays a significant role in determining the dephasing length. This may help in maintaining the desired power level required for the laser fusion schemes when the target is placed at a large distance. Figure 5 also suggests that in the absence of an external magnetic field ($\omega_c = 0$), L_d is greater for the PSP than for the FSP. Figure 6 shows the numerical solution of equation (29) for the GVD coefficient GVD as a function of the external magnetic field ratio for different values of the relativistic factor. We have plotted GVD versus ω_c/ω_p with and without the last term of equation (29) for the PSP and the FSP, respectively. The rate of increase of GVD for PSP is greater than that for the FSP for a given value of ω_c/ω_p. This in turn supports the growth rate of the AMI and the self-phase modulation instability. It is also noted that GVD for the FSP is less than that for the PSP in the absence of an external magnetic field ($\omega_c = 0$).

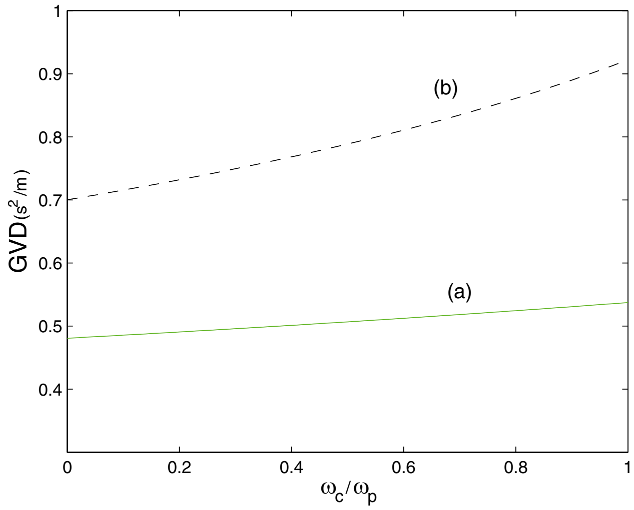

Figure 6. Variation of the GVD ($=\partial^2 k_0/\partial\omega_0^2$) with the magnetic field ratio ω_c/ω_p for $\omega_p = 2.8 \times 10^{13}$ rad s^{-1} for (a) FSP and (b) PSP. At $\omega_c = 0$, L_d for PSP is greater than that for FSP

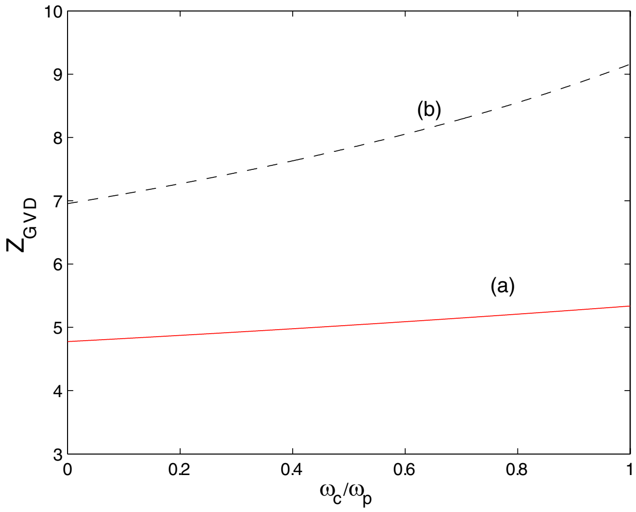

Figure 7. Variation of the GVD length Z_{GVD} with the magnetic field ratio ω_c/ω_p for $\omega_p = 2.8 \times 10^{13}$ rad s^{-1} for (a) FSP and (b) PSP. At $\omega_c = 0$, L_d for PSP is greater than that for FSP.

Figure 7 shows the numerical solution of equation (30) for the dispersion length Z_{GVD} as a function of the external magnetic field ratio. We have plotted Z_{GVD} versus ω_c/ω_p with and without the last term of equation (30) for the PSP and the FSP, respectively. We noted that the dispersion length Z_{GVD} increases with ω_c/ω_p. The dispersion growth rate for PSP is higher than that for the FSP for a given value of the magnetic field ratio. Figure 7 also suggests that the value of Z_{GVD} for PSP is greater than that for the FSP in the absence of an external magnetic field ($\omega_c = 0$).

5. Conclusions

The present work deals with the propagation of a relativistic intense short laser pulse through a partially stripped magnetoplasma channel in the presence of an external static longitudinal magnetic field. The polarizability of the bound electrons and the external magnetic field significantly change the evolution and interaction characteristics of the pulse. Three-dimensional coupled equations are derived for describing the evolution of the pulse in the partially stripped magnetoplasma channel. The excitation conditions and the growth rate of the modulation instability were deduced. It was observed that the growth rate of the modulation instability increases with the external magnetic field and can reach a value higher than the maximum growth rate of the forward Raman scattering instability. It is also observed that the modulation instability could only be excited in a given range of laser intensity and frequency. In addition to this, it is also observed that both the bound electrons and the external magnetic field significantly contribute to the growth of the positive chirp of the laser frequency and intensity perturbation. This study reveals that the presence of the bound electrons and that of an external magnetic field lead to a significant change in the self-phase modulation, yielding a significant change in the growth rate of the atomic modulation instability. The relation for the group velocity dispersion and the dephasing length is also formulated and analyzed numerically. Finally, we pointed out that the existence of the magnetic field and that of the bound electrons lead to significant enhancement of the dispersion length and the dephasing length.

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